

NUMERICAL METHOD OF INVESTIGATING
NONSTATIONARY SPATIAL MOTIONS OF A
COMPRESSIBLE GAS

G. M. Makhviladze and S. B. Shcherbak

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A method is suggested for numerical integration of nonstationary spatial Navier-Stokes equations describing the motion of a compressible heat-conducting gas. As an example, the numerical solution is provided of the two-dimensional natural convection problem of a compressible gas in a closed volume.

1. The necessity of numerical solution of non-one-dimensional Navier-Stokes equations for a compressible viscous, heat-conducting gas is generated in studying various physical processes which are important in present-day technology. An important problem is the creation of "faster" calculation methods.

Various finite-difference schemes were developed, explicit and implicit (their characteristics and bibliography can be found, e.g., in [1-4]). The explicit ones are simple and economical, since they require a minimal number of operations at each time layer. However, to guarantee stability of these schemes it is necessary to impose restrictions on the time step (or the iteration parameter in the case of stationary problems) with which the calculation is performed:

$$\tau \leq \min \{ \tau_1, \tau_2, \tau_3, \tau_s \}, \quad (1)$$

$$\tau_1 = h/u, \quad \tau_2 = h^2/\kappa, \quad \tau_3 = h^2/\nu, \quad \tau_s = h/c_s.$$

The stability conditions (1) can significantly decrease the effectiveness of calculation due to the necessity of selecting a time step significantly smaller than dictated by the physical process itself. If, e.g., the motion is essentially subsonic, which is characteristic of many problems of combustion, convection, and other problems of thermodynamics, the restriction $\tau \leq \tau_s = h/c_s$ can be purely formal from the physical point of view, related only to the choice of the finite-difference scheme. Thus, in combustion problems the characteristic gas velocities are $U \lesssim 10$ m/sec, while the sound velocity is $c_s \gtrsim 300$ m/sec, i.e., for the ratio we have $c_s/U \gtrsim 30$. Consequently, the calculation time of such "slow" problems can be extended by at least 30 times due to the restriction on the time step $\tau \leq \tau_s$ imposed by the choice of an explicit scheme.

Implicit finite-difference schemes of intermediate type (between explicit and purely implicit) have become most popular, as they allow to avoid some of the restrictions on the time step or reduce them by an appreciable factor. Along with the less rigid stability conditions these schemes, constructed by means of the fractional step method [1, 4], lead to a number of operations per time step comparable with the number of operations for explicit schemes.

Several such schemes [5-11] were suggested and tried for solving the full two-dimensional nonstationary Navier-Stokes equations of a compressible gas.

The study of various natural convection processes became possible due to the scheme suggested by Polezhaev [5] (see also [6]). The scheme of [5] is conditionally stable: it has no restriction on the step $\tau \leq \{ \tau_2, \tau_3 \}$, but requires satisfaction of the condition $\tau \leq \tau_* \sim \tau_s$ (τ_* is the maximum allowed step in [5]), due, obviously, to approximating pressure terms on the lower time layer. It must be noted, however, that the remaining stability condition $\tau \leq \tau_* \sim \tau_s$ in studying slow motions can provide a strong restriction on the time step and lead to a large expenditure in computer time. We have encountered this fact, rendering the analysis difficult, in using the scheme of [5] for solving a problem in combustion theory.

Implicit schemes, possessing a significant stability margin, were proposed [7-10] for obtaining station-

ary solutions. These schemes, in which τ is used as an iteration parameter, make it possible to obtain a stationary solution with a small expenditure in computer time. However, in calculating nonstationary processes with a time step $\tau > \tau_*$ they provide a solution differing from the true one. Thus, a detailed comparison of the schemes of [5] and [7], carried out in [11] in solving the two-dimensional convection problem, showed that by the scheme of [7] the stationary solution can be obtained 3 times faster than by using the scheme of [5]. The nonstationary process of establishing stationary gas motion by [7] was calculated correctly only by using the same time step $\tau = \tau_*$, as in the calculations of [5]. For many problems it is of interest to consider the nonstationary period of establishing stationary motion. It must also be taken into account that a stationary solution may not exist or may not be unique. In these cases an effective calculation of the nonstationary process is necessary.

In the present paper we construct an implicit finite-difference scheme over the coordinates of a division, possessing the properties of the total approximation. Using it, one can integrate numerically the nonstationary Navier-Stokes equations describing spatial motion of a compressible gas with a rate dictated by the speed of calculating the physical process. The scheme suggested is effective in calculating both stationary and nonstationary processes, and gives a significant gain in time. To illustrate its possibilities we provide a numerical solution of a model problem on natural convection of a compressible gas in a closed vessel whose walls are supported at different temperatures, and compare the results with those provided by the scheme of [5].

2. In matrix notation the system of nonstationary, two-dimensional Navier-Stokes equations describing the motion of a viscous, heat-conducting compressible gas is

$$\frac{\partial \varphi}{\partial t} + A(\varphi) \varphi = F, \quad A = A_1 + A_2 + A_3, \quad (2)$$

$$\varphi = \begin{pmatrix} u \\ v \\ P \\ T \end{pmatrix}, \quad A_1 = \begin{pmatrix} u \frac{\partial}{\partial x} - \frac{4}{3 \text{Re} \rho} \frac{\partial^2}{\partial x^2} & 0 & \frac{1}{\gamma M^2 \rho} \frac{\partial}{\partial x} & 0 \\ 0 & u \frac{\partial}{\partial x} - \frac{1}{\text{Re} \rho} \frac{\partial^2}{\partial x^2} & 0 & 0 \\ \gamma P \frac{\partial}{\partial x} & 0 & u \frac{\partial}{\partial x} & -\frac{\gamma}{\text{Re Pr}} \frac{\partial^2}{\partial x^2} \\ (\gamma - 1) T \frac{\partial}{\partial x} & 0 & 0 & u \frac{\partial}{\partial x} - \frac{\gamma}{\text{Re Pr} \rho} \frac{\partial^2}{\partial x^2} \end{pmatrix},$$

$$A_2 = \begin{pmatrix} v \frac{\partial}{\partial y} - \frac{1}{\text{Re} \rho} \frac{\partial^2}{\partial y^2} & 0 & 0 & 0 \\ 0 & v \frac{\partial}{\partial y} - \frac{4}{3 \text{Re} \rho} \frac{\partial^2}{\partial y^2} & \frac{1}{\gamma M^2 \rho} \frac{\partial}{\partial y} & 0 \\ 0 & \gamma P \frac{\partial}{\partial y} & v \frac{\partial}{\partial y} & -\frac{\gamma}{\text{Re Pr}} \frac{\partial^2}{\partial y^2} \\ 0 & (\gamma - 1) T \frac{\partial}{\partial y} & 0 & v \frac{\partial}{\partial y} - \frac{\gamma}{\text{Re Pr} \rho} \frac{\partial^2}{\partial y^2} \end{pmatrix},$$

$$A_3 = \begin{pmatrix} 0 & -\frac{1}{3 \text{Re} \rho} \frac{\partial^2}{\partial x \partial y} & 0 & 0 \\ -\frac{1}{3 \text{Re} \rho} \frac{\partial^2}{\partial x \partial y} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad F = \begin{pmatrix} -\text{Fr}_x^{-1} \\ -\text{Fr}_y^{-1} \\ 0 \\ 0 \end{pmatrix}.$$

Here A_1 acts in the x direction; A_2 in the y direction; and the operator A_3 contains mixed derivatives. The values of the dimensionless parameters M , Re , Pr , Fr_x , and Fr_y for each specific problem are determined by the choice of characteristic quantities, length, time, density, and temperature, to which the dimensional quantities refer. The gas is assumed to be ideal. The viscosity and heat-conduction coefficients are assumed constant, and viscous energy dissipation is not taken into account. The pressure equation was obtained by combining the continuity and temperature equations, multiplying them by T and ρ , respectively.

We introduce a uniform space-time grid $x_i = ih$, $y_j = jl$, $t_n = n\tau$, and denote $\varphi(t_n, x_i, y_j) = \varphi_{i,j}^n$. The time and space derivatives are approximated as follows:

$$\frac{\partial \varphi}{\partial t} \sim \frac{\varphi^{n+1} - \varphi^n}{\tau}, \quad \frac{\partial \varphi}{\partial x} \sim \frac{\varphi_{i+1} - \varphi_{i-1}}{2h}, \quad \frac{\partial^2 \varphi}{\partial x^2} \sim \frac{\varphi_{i+1} - 2\varphi_i + \varphi_{i-1}}{h^2}. \quad (3)$$

Relating the space derivatives to the upper time layer, we obtain a grid equation corresponding to Eq. (2):

$$\frac{\varphi^{n+1} - \varphi^n}{\tau} + \hat{A}(\varphi^{n+1}) \varphi^{n+1} = F^{n+1} \quad (\hat{A} = \hat{A}_1 + \hat{A}_2 + \hat{A}_3). \quad (4)$$

Let $\tilde{\varphi}^{n+1}$ be the solution on the $(n+1)$ -th layer, constructed by linear extrapolation from the known values of the original function onto two later time layers:

$$\tilde{\varphi}^{n+1} = \varphi^n + \Delta^n, \text{ where } \Delta^n = \varphi^n - \varphi^{n-1} \quad (5)$$

is the increment of the original function on neighboring time layers.

We introduce the quantity

$$\varepsilon^{n+1} = \varphi^{n+1} - \tilde{\varphi}^{n+1}, \quad (6)$$

the deviation between the unknown function and its preliminary constructed value (Δ and ε are columns with components $\Delta^u, \Delta^v, \Delta^p, \Delta^t$ and $\varepsilon^u, \varepsilon^v, \varepsilon^p, \varepsilon^t$, respectively).

Substituting (5) and (6) into (4) and replacing $\hat{A}(\varphi^{n+1})$ by $\hat{A}(\tilde{\varphi}^{n+1})$, which causes an insignificant error in the case of small ε , we obtain

$$\{E + \tau[\hat{A}_1(\tilde{\varphi}^{n+1}) + \hat{A}_2(\tilde{\varphi}^{n+1})]\} \varepsilon^{n+1} + \tau \hat{A}_3(\tilde{\varphi}^{n+1}) \varepsilon^{n+1} = -\Delta^n + \tau F^{n+1} - \tau \hat{A}(\tilde{\varphi}^{n+1}) \tilde{\varphi}^{n+1}. \quad (7)$$

An effective numerical realization of this equation can be obtained by discarding the term $\tau \hat{A}_3(\tilde{\varphi}^{n+1}) \varepsilon^{n+1}$. Indeed, replacing the operator in the left-hand side of Eq. (7) by the factorization

$$[E + \tau(\hat{A}_1 + \hat{A}_2)] \varepsilon \approx (E + \tau \hat{A}_1)(E + \tau \hat{A}_2) \varepsilon, \quad (8)$$

we obtain the equation

$$[E + \tau \hat{A}_1(\tilde{\varphi}^{n+1})][E + \tau \hat{A}_2(\tilde{\varphi}^{n+1})] \varepsilon^{n+1} = -\Delta^n + \tau F^{n+1} - \tau \hat{A}(\tilde{\varphi}^{n+1}) \tilde{\varphi}^{n+1}, \quad (9)$$

whose solution is equivalent to the solution of the one-dimensional linear equations with block matrices:

$$[E + \tau \hat{A}_1(\tilde{\varphi}^{n+1})] \xi = -\Delta^n + \tau F^{n+1} - \tau \hat{A}(\tilde{\varphi}^{n+1}) \tilde{\varphi}^{n+1}, \quad (10)$$

$$[E + \tau \hat{A}_2(\tilde{\varphi}^{n+1})] \varepsilon^{n+1} = \xi, \quad (11)$$

where ξ is a column with components $\xi^u, \xi^v, \xi^p, \xi^t$ (for a more detailed description with the use of grid subscripts see [12]).

Equation (10) with fourth-order matrices serves for determining the auxiliary quantity ξ . The structure of the equations, however, is such that to solve it one needs a matrix displacement along the x axis with third-order matrices only. The equation for ξ^v is solved by scalar displacement along the same direction. Following this, the quantity ε^{n+1} is similarly determined from Eq. (11) by displacements along the y axis. By Eq. (6) we then calculate the unknown quantity φ^{n+1} . The suggested difference scheme with the total approximation is of order $O(\tau + h^2 + l^2)$. The stationary solution can be obtained by establishing an asymptotic solu-

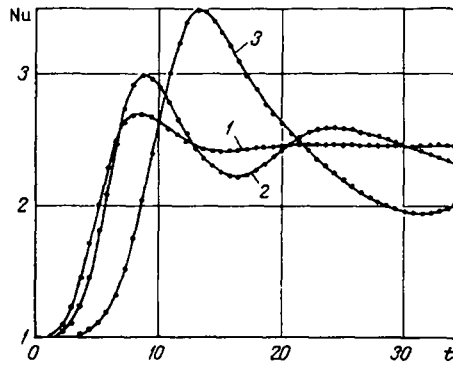


Fig. 1

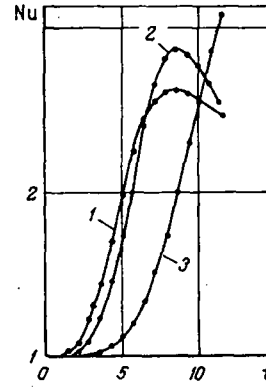


Fig. 2

Fig. 1. Time dependence of the Nusselt number for a heated wall, obtained by the scheme of [5] with a step $\tau = \tau_* = 0.0056$ and by the scheme of Sec. 2 with $\tau = 2\tau_*$, $4\tau_*$, $8\tau_*$ (curve 1), $16\tau_*$ (2), $32\tau_*$ (3) ($M = M_1$, 21×21 grid).

Fig. 2. $Nu(t)$, calculated by the scheme of [5] with $\tau = \tau_* = 0.0028$, and by the scheme of Sec. 2 with $\tau = 8\tau_*$, $16\tau_*$ (curve 1), $32\tau_*$ (2), $64\tau_*$ (3) ($M = M_1$, mesh 41×41).

tion of the nonstationary problem at long times.

Linear stability analysis of the scheme suggested was performed for simple two-dimensional model equations, the heat-conduction equation, the "sound" kernel of the Navier-Stokes system of equations, and the equations of convective transport. It seems that for the first two cases the scheme is absolutely stable. In the third case the stability condition is $\tau \leq \tau_1$, i.e., there is no restriction on the time step $\tau \leq \tau_S$. Undoubtedly, this analysis is incomplete, and the stability of the method can be judged only by numerical solution of the full equations. In systematic calculations (see Sec. 3) the scheme was stable up to Courant numbers $K \leq 10^4$ ($K = \tau/hM$), which in dimensional variables corresponds to $\tau \leq 10^4 \tau_S$ (calculations were not carried out for large K values).

The possibilities of the method suggested were analyzed on the model problem of convection of a compressible gas in a closed region [6].

3. Consider the motion of a compressible gas in a closed rectangular region $0 \leq x \leq H_0$, $0 \leq y \leq L_0$ in the presence of a gravity force along the vertical axis ($Fr_x^{-1} = 0$). On the side walls $x = 0$, $0 \leq y \leq L_0$ and $x = H_0$, $0 \leq y \leq L_0$ there are constant temperature values ($T_2 > T_1$), and the top and bottom are thermally isolated ($\partial T/\partial y = 0$). At the walls the sticking condition $u = v = 0$ is satisfied for $x = 0$, $x = H_0$ and $y = 0$, $y = L_0$.

As initial conditions, we assign linear temperature and gas flow distributions

$$u(0, x, y) = v(0, x, y) = 0, T(0, x, y) = T_2 - (T_2 - T_1)x/H_0, P = \text{const}, \quad (12)$$

corresponding to the stationary state of the system in the absence of gravity.

To set up the problem in terms of characteristic scales used in introducing dimensionless variables, we take the quantities L_0 , T_1 , ρ_1 , and $\sqrt{L_0/g}$ (the velocity scale is then the quantity $\sqrt{L_0 g}$).

The dimensionless numbers appearing in the original equations acquire the form

$$M = \sqrt{\frac{L_0 g}{\gamma R T_1}}, \text{Re} = \frac{L_0 \sqrt{L_0 g} \rho_1}{\eta}, Fr_y = 1 \quad (13)$$

(the corresponding Rayleigh number is expressed by the equation $Ra = Pr \text{Re}^2 (T_2 - T_1)/T_1$).

The results of calculations are conveniently characterized by the time behavior of the mean Nusselt number

$$Nu(x) = \frac{H}{T_a} \int_0^1 \frac{\partial T}{\partial x} dy \left(H = \frac{H_0}{L_0}, T_a = \frac{T_2}{T_1} - 1 \right). \quad (14)$$

Initially one usually assigns $\Delta^u = \Delta^v = \Delta^P = \Delta^T = 0$.

To control the accuracy of the calculations, we incurred mass balance at the nonstationary portion, while in the stationary region the following relations are satisfied

$$\text{Nu}(0) = \text{Nu}(H), \int_0^1 \int_0^H \rho u dx dy = 0, \int_0^1 \int_0^H \rho v dx dy = 0. \quad (15)$$

In realizing the numerical algorithm we used as boundary conditions for P the original pressure equation, written with account of the boundary conditions for velocity and temperature. In this case the first spatial derivatives were approximated by the three-point equation of second-order accuracy, and the second derivatives were taken over the normal to the wall by a first-order accuracy equation. The separation of boundary conditions for velocity and temperature does not require explanation; the boundary conditions for pressure were also separated, as inside the region.

Each variant was calculated by two methods: by the scheme discussed in Sec. 2, and by the scheme of [5]. In the calculations we used the following values of dimensionless parameters: $\gamma = 1.4$, $H = 1$, $\text{Re} = 250$, $\text{Pr} = 0.71$, $T_a = 0.5$ ($\text{Ra} \approx 2.2 \cdot 10^4$). The calculations were performed with various time steps over a 21×21 grid for values of the Mach number $M_1^2 = 1/320$, $M_2^2 = 1/20$. Several results were obtained on a 41×41 grid. Programs were written in FORTRAN-IV for an EC-1040 computer. Results of calculations are presented in Figs. 1-3.

Figure 1 shows $\text{Nu}(t)$, obtained by the scheme of [5] with the maximum step allowed for it $\tau_* = 2\tau_S = 2hM_1$, and by the scheme of Sec. 2. The difference in the Nu values, obtained by the scheme of [5] and by the scheme of (10), (11) with $\tau = 2\tau_*$ and $4\tau_*$, consists of 0.3%, and with $\tau = 8\tau_* - 1.5\%$, i.e., the results practically coincide, therefore they are represented on the figure by one line (curve 1).

Thus, an eightfold increase in the step gives fully satisfactory results. Further increase in the time step leads to a significant difference in the Nu values (curves 2 and 3).

Similar results were also obtained for Mach number values $M = M_2$.

For $M = M_1$ we also performed calculations on a fine grid (Fig. 2). Results of calculations by the scheme of [5] with the maximum allowed step for it τ_* and by the scheme (10), (11) with $\tau = 8\tau_*$, $16\tau_*$ lie on the same curve 1. The difference in Nu values, calculated by the scheme of [5] and by the scheme of Sec. 2 with a step 16 times larger, is less than 1.5%. The general shape of the dependence practically does not change in comparison with Fig. 1. Thus, in using scheme (10), (11) the size reduction in the spatial step does not lead to a decrease in the optimum time step, with which one can calculate a nonstationary process quite accurately. The calculation time up to $t = 11.4$ (further calculations were not performed) was 5.5 h by the scheme of [5], and 30 min by the scheme of Sec. 2 with $\tau = 16\tau_*$.

In all calculations performed the mass balance was satisfied with an accuracy of 0.5%. Relations (15) were satisfied with an accuracy of 0.5, 3, and 3%, respectively.

Methodically performed experiments indicate the effectiveness of the calculation method suggested for solving both stationary and nonstationary problems. For the convection model problem a significant gain in calculation time was obtained in comparison with the method of [5].

4. We have a number of remarks concerning the method suggested. This method can be generalized and used for constructing a whole scheme of problems approximated by nonlinear equations in partial derivatives. We discuss the general idea of their construction on the example of the grid equation

$$\frac{\varphi^{n+1} - \varphi^n}{\tau} + \hat{A}(\varphi^{n+1}) \varphi^{n+1} = 0, \hat{A} = \hat{A}_1 + \hat{A}_2, \quad (16)$$

in which \hat{A}_1 and \hat{A}_2 are grid operators, each of which acts only in one direction.

Let $\tilde{\varphi}^{n+1}$ be an approximate solution on the $(n+1)$ -th layer by any method. We introduce, as above, the quantities

$$\varepsilon^{n+1} = \varphi^{n+1} - \tilde{\varphi}^{n+1}, \Delta^{n+1} = \varphi^{n+1} - \varphi^n. \quad (17)$$

Substituting (17) into (16) and replacing $\hat{A}_i(\varphi^{n+1})$ by $\hat{A}_i(\tilde{\varphi}^{n+1})$, we obtain

$$\varepsilon^{n+1} + \tau [\hat{A}_1(\tilde{\varphi}^{n+1}) + \hat{A}_2(\tilde{\varphi}^{n+1})] \varepsilon^{n+1} = -(\tilde{\varphi}^{n+1} - \varphi^n) - \tau [\hat{A}_1(\tilde{\varphi}^{n+1}) + \hat{A}_2(\tilde{\varphi}^{n+1})] \tilde{\varphi}^{n+1}. \quad (18)$$

We construct a scheme similar to (6)-(11):

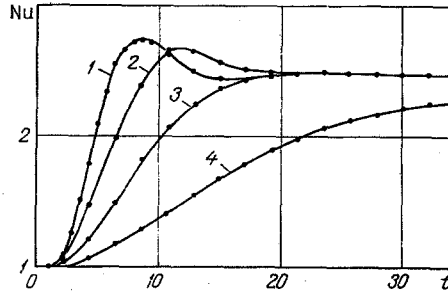


Fig. 3. $Nu(t)$, calculated by the scheme of [5] with $\tau = \tau_*$ (curve 1), and by the scheme of Eq. (20) with $\tau = 4\tau_*$ (2), $8\tau_*$ (3), $16\tau_*$ (4) ($M = M_2$, mesh 21×21)

$$[E + \tau \hat{A}_1(\tilde{\varphi}^{n+1})] \xi = -(\tilde{\varphi}^{n+1} - \varphi^n) - \tau \hat{A}(\tilde{\varphi}^{n+1}) \tilde{\varphi}^{n+1}, \quad (19)$$

$$[E + \tau \hat{A}_2(\tilde{\varphi}^{n+1})] \varepsilon^{n+1} = \xi.$$

The effectiveness of the latter depends on how well the function $\tilde{\varphi}^{n+1}$ was selected.

At the first stage of study we tried the simplest scheme, in which a value φ^n was selected as the function $\tilde{\varphi}^{n+1}$. In this case the quantity $\varepsilon^{n+1} = \varphi^{n+1} - \varphi^n$, on which the operator \hat{A} acts, was projected on the upper time layer, is the difference of values of the unknown functions on neighboring time layers. Equation (19) acquires the form

$$[E + \tau \hat{A}_1(\varphi^n)] \xi = -\tau \hat{A}(\varphi^n) \varphi^n, \quad [E + \tau \hat{A}_2(\varphi^n)] \varepsilon^{n+1} = \xi. \quad (20)$$

The results of solving the convection test problem, considered in Sec. 3, by means of Eq. (20) are shown on Fig. 3. The scheme of (20) gives correct results on the nonstationary portion only in calculations with $\tau \leq \tau_*$. For large τ the solution on the nonstationary portion deviates more strongly from curve 1. Nevertheless Eq. (20) is useful for obtaining a stationary solution with large τ .

We note that the scheme of (20) is similar to the scheme suggested in [7] (see also [10, 11]). The difference consists only of the fact that in (20) the operator \hat{A} is decomposed into two operators: \hat{A}_1 and \hat{A}_2 , acting in different directions, while in [7] it is decomposed into four (along with decomposition in coordinates, one involving physical processes was also used in [7]).

Since the scheme of (20) is insufficiently effective in calculating a nonstationary process, we constructed the scheme (10), (11), discussed in Sec. 2, with $\tilde{\varphi}^{n+1} = \varphi^n + \Delta^n$, giving quite good results.

The procedure of constructing decomposition schemes can be continued. The following step, e.g., consists of calculating $\tilde{\varphi}^{n+1}$ by the values of the unknown function at the latter 3 time layers by means of quadratic extrapolation, i.e., $\tilde{\varphi}^{n+1} = \varphi^n + \Delta^n + \varepsilon^n$.

Calculations by Eq. (19) with the functions $\tilde{\varphi}^{n+1}$ thus determined, performed for the convection problem formulated in Sec. 3, gave results practically coinciding with those obtained by means of the basic scheme (10), (11).

In a number of systematic experiments on the test scheme (10), (11), as well as (20) and the scheme with $\tilde{\varphi}^{n+1} = \varphi^n + \Delta^n + \varepsilon^n$ applied to the problem of Sec. 3, the time derivative was approximated by the second-order equation $\partial\varphi/\partial t \sim (3\varphi^{n+1} - 4\varphi^n + \varphi^{n-1})/2\tau$, which did not lead, however, to any change in the results given above. This implies that the error in calculations with a step $\tau > \tau_*$ is, obviously, determined basically by the decomposition error.

In conclusion, we note the general nature of the method of calculation suggested. It can be considered as a distinct three-layer iteration method for solving systems of nonlinear equations in partial derivatives (not necessarily the Navier-Stokes equations). Recently performed calculations of convection problems by means of the method discussed above showed its effectiveness in using nonuniform grids in solving equations with three spatial variables. Further study of the possibilities of the method suggested, as well as its analytic justification, are necessary.

NOTATION

τ , time step; h , spatial step; $U(u, v)$, gas velocity; κ , thermal diffusivity; ν , kinematic viscosity; c_s , sound velocity; φ , unknown vector function; A , a nonlinear differential operator matrix; t , time; x and y , Cartesian coordinates; ρ , density; P , pressure; T , temperature; γ , adiabatic index; M , Mach number; Re , Reynolds number; Pr , Prandtl number; Ra , Rayleigh number; Nu , Nusselt number; Fr_x and Fr_y , Froude numbers; \hat{A} , grid operator; E , unity operator; K , Courant number; H_0 and L_0 , sizes of the calculation domain; T_1 and T_2 , side wall temperatures; ρ_1 , gas density at the cold wall; g , gravitational acceleration; R , gas constant; and η , dynamic viscosity.

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EFFECT OF CARBON ON HEAT TRANSFER THROUGH A PISTON OF AN INTERNAL COMBUSTION ENGINE

S. I. Sevast'yanov

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By analyzing and correlating the results of thermometrization of a 11D45 diesel we estimate the effect of carbon in oil-cooling channels on parameters for heat transfer through a piston.

When boosted diesel locomotives operate on group B motor oils (M-12B, M-14B, etc.), carbon deposits form on cylinder and piston components. Observations show that carbon deposits in the inner cavity of an oil-cooled piston head are particularly harmful and dangerous. Heavy carbon deposits and ineffective decrustation in the oil-cooling region shorten the useful life of pistons in 2D100, 10D100, 11D45, etc. diesels by a factor of three to four [1, 2].

It is well known that carbon deposits have an appreciable effect on heat transfer, yet in treatises on heat transfer in internal combustion engines the effect of carbon deposits is generally ignored. This naturally hampers their use in practical problems of increasing the reliability of diesel locomotives.

On the basis of analysis and correlation of the results of thermometrization of a 11D45 diesel we estimate the effect of carbon on heat transfer through a piston [3-5].

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